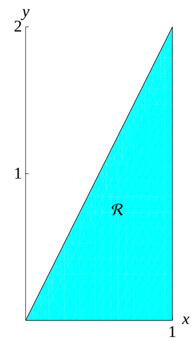
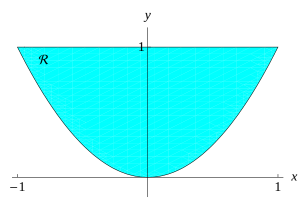
# Double Integrals over General Regions

1. Let $\mathcal{R}$ be the region in the plane bounded by the lines $y = 0$, $x = 1$, and $y = 2x$. Evaluate the double integral $\iint_{\mathcal{R}} 2xy \, dA$.

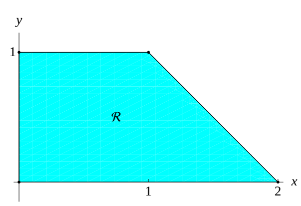



2. Let $\mathcal{R}$ be the region bounded by $y = x^2$ and $y = 1$. Write the double integral $\iint_{\mathcal{R}} f(x, y) \, dA$ as an iterated integral in both possible orders.



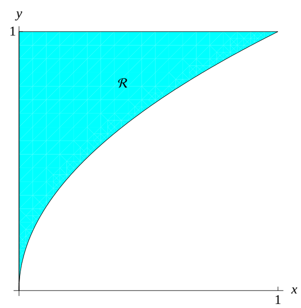

3. *For many regions, one order of integration will be simpler to deal with than the other. That is the case in this problem: use the shape of the region to decide which order of integration to use. Why is the other order more difficult?*

   Let $\mathcal{R}$ be the trapezoid with vertices $(0, 0)$, $(2, 0)$, $(1, 1)$, and $(0, 1)$. Write the double integral $\iint_{\mathcal{R}} f(x, y) \, dA$ as an iterated integral.



4. *Sometimes, when converting a double integral to an iterated integral, we decide the order of integration based on the integrand, rather than the shape of the region — some integrands are easy to integrate with respect to one variable and much harder (or even impossible) to integrate with respect to the other. That is the case in this problem.*

   Evaluate the double integral $\iint_{\mathcal{R}} \sqrt{y^3+1}\, dA$ where $\mathcal{R}$ is the region in the first quadrant bounded by $x = 0$, $y = 1$, and $y = \sqrt{x}$. (To decide the order of integration, first think about whether it's easier to integrate the integrand with respect to $x$ or with respect to $y$.)




5. In each part, you are given an iterated integral. Sketch the region of integration, and then change the order of integration.

   (a) $\int_0^4 \int_0^x f(x,y)\ dy\ dx.$

   (b) $\int_0^4 \int_0^{\sqrt{y}} f(x,y)\ dx\ dy.$

   (c) $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\ dx\ dy.$

6. Let $a$ be a constant between 0 and 4. Let $\mathcal{R}$ be the region bounded by $y = x^2 + a$ and $y = 4$. Write the double integral $\iint_{\mathcal{R}} f(x, y)\, dA$ as an iterated integral in both possible orders.

7. Evaluate the iterated integral $\displaystyle\int_0^1 \int_{-\sqrt{1-x^2}}^0 2x \cos\left(y - \frac{y^3}{3}\right)\, dy\, dx$.

More problems on the other side!

8. A flat plate is in the shape of the region in the first quadrant bounded by $x = 0$, $y = 0$, $y = \ln x$ and $y = 2$. If the density of the plate at point $(x, y)$ is $xe^y$ grams per $\text{cm}^2$, find the mass of the plate. (Suppose the $x$- and $y$-axes are marked in cm.)

9. Let $\mathcal{U}$ be the solid above $z = 0$, below $z = 4 - y^2$, and between the surfaces $x = \sin y - 1$ and $x = \sin y + 1$. Find the volume of $\mathcal{U}$.